Back Cast of Interim Solution B+ to Improve Real-Time Scarcity Pricing

Whitepaper

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1. Executive Summary

1.1. Scope

The Public Utility Commission of Texas (PUCT) has requested that ERCOT perform a back cast of an interim proposal that will approximate Real-Time co-optimization of energy and Ancillary Services (AS). This interim proposal has been described as the "Interim Solution B+" and is intended to be a more appropriate method of pricing scarcity during conditions of low operating reserves in Real-Time. This back cast approximates the pricing outcomes and estimates what the market impacts may have been if "Interim Solution B+" had been in place for the years 2011 and 2012. This analysis builds off of the previous "Interim Solution B" back cast that was filed by ERCOT on February 13, 2013.

1.2. Background

The concept of the "Interim Solution B+" was initiated by a paper by William Hogan, "Electricity Scarcity Pricing through Operating Reserves: An ERCOT Window of Opportunity,"[1] which was filed with the PUCT by GDF Suez on November 14, 2012. The paper emphasized the importance of an Operating Reserve Demand Curve (ORDC) in improving Real-Time scarcity pricing in the ERCOT market. The proposed approach involves the Real-Time co-optimization of energy and AS.

Preliminary analysis of the timeframe for implementing Real-Time co-optimization of energy and AS indicated that it could not be done quickly. ERCOT contacted Professor Hogan to determine the validity of modifying the existing Energy Offer floors as an interim solution. This approach was labeled as "Interim Solution A." The resulting collaboration produced a calculation based on Loss of Load Probability (LOLP), Value of Lost Load (VOLL) and the level of available reserves in Real-Time, which was labeled as "Interim Solution B." Both interim solutions were presented and discussed at a PUCT workshop held on January 24, 2013.

During the January 24, 2013 workshop, concerns were raised about "Interim Solution B" in terms of negative market behavior that the proposal could incentivize. A modified proposal addressed the incentive issues in an "Interim Solution B+" approach by adding an AS imbalance settlement to the "Interim Solution B" approach. This whitepaper first introduces the concept of Real-Time co-optimization of energy and AS with an ORDC, and then describes "Interim Solution B+," which is an approximation to this concept.

1.3. Summary of Back Cast Results

The "Interim Solution B+" is an approximation to a full Real-Time energy and AS co-optimization solution. In this approximation, a price adder for energy is calculated on top of the original energy price which is intended to capture the value of the opportunity cost of reserves.

[1] W. Hogan, "Electricity Scarcity Pricing through Operating Reserves: An ERCOT Window of Opportunity," Mossavar-Rahmani Center for Business and Government, Harvard Kennedy School, November 1, 2012, available at http://www.hks.harvard.edu/fs/whogan/Hogan_ORDC_110112r.pdf .

In addition, there is an AS imbalance settlement which makes Resources indifferent to the utilization of their capacity for energy or reserves. The back cast results vary significantly with different parameters for the ORDC and with the VOLL at different future System-Wide Offer Caps (SWCAPs). Pending the results of ongoing studies to estimate VOLL, the values utilized here reflect the range of generation offer caps.

The back cast analysis of the price adder shows that the energy-weighted average energy price increases over a range of $7/MWh to $26.08/MWh in 2011 and $1.08/MWh to $4.5/MWh in 2012. This range results from different parameter settings that were used in the back cast. The back cast results for the average energy price increase with minimum contingency levels (X) of 1375 MW and 1750 MW are presented in Table 1. At the minimum contingency level, scarcity prices achieve the maximum allowed value.

Table 1 : Energy-weighted average energy price adder (and Online reserve price) ($/MWh) for 2011 & 2012 for different VOLLs and minimum contingency levels (X)

VOLL	Energy-weighted average price increase with X at 1375 MW ($/MWh)			Energy-weighted average price increase with X at 1750 MW ($/MWh)		
	2011	2012	2011 & 2012 combined	2011	2012	2011 & 2012 combined
$5000/MWh	7.00	1.08	4.08	12.03	2.40	7.28
$7000/MWh	11.27	1.56	6.48	19.06	3.45	11.35
$9000/MWh	15.54	2.05	8.87	26.08	4.50	15.42

Due to the increase in energy prices resulting from the proposal, the potential impacts on Peaker Net Margin (PNM) were also analyzed. The additional PNM from implementing "Interim Solution B+" is presented in Table 2 for different VOLLs and minimum contingency levels (X). For the purpose of comparison, a study was also performed to determine the potential impacts to 2011 and 2012 of simply having the SWCAP set to higher values. Table 3 presents the estimates of additional PNM that may have been observed by solely increasing SWCAP to the different VOLLs being used in the back cast analysis.

Table 2 : Estimated additional PNM ($/MW) from "Interim Solution B+" for 2011 & 2012 for different VOLLs and minimum contingency levels (X)

VOLL	Total Additional PNM under Interim Solution B+ with X at 1375 MW ($/MW)		Total Additional PNM under Interim Solution B+ with X at 1750 MW ($/MW)	
	2011	2012	2011	2012
$5000/MWh	38,544	7,740	67,892	17,267
$7000/MWh	62,141	11,189	107,327	24,809
$9000/MWh	85,773	14,643	146,795	32,362

Table 3 : Estimated additional PNM ($/MW) for 2011 and 2012 by only increasing the SWCAP

SWCAP	Total Additional PNM if SWCAP Increased to VOLL ($/MW)	
	2011	2012
$5000/MWh	57,631	2,877
$7000/MWh	114,168	5,883
$9000/MWh	170,706	8,889

As part of the "Interim Solution B+" proposal, a Real-Time AS imbalance settlement is introduced. This is intended to account for the fact that Resources may have a different amount of reserves available in Real-Time relative to the amount that they were obligated to provide based on activities in the Day-Ahead Market (DAM) and Adjustment Period. This can result in Qualified Scheduling Entities (QSEs) needing to purchase reserves in Real-Time to cover those responsibilities. The AS imbalance settlement analysis shows a net refund to loads, which ranges from $60.2M to $218.4M in 2011 and $1.6M to 4.3M in 2012. Of the $218.4M in 2011, $214M is from the extreme weather that occurred in February and August. Table 4 summarizes these back cast results. The positive sign for the values in the table indicates a net charge to Resources.

Table 4 : Net AS imbalance settlement ($) charge to Resources for 2011 & 2012 for different VOLLs and minimum contingency levels (X)

VOLL	Net AS Imbalance Settlement for All Reserves with X at 1375 MW ($)			Net AS Imbalance Settlement for All Reserves with X at 1750 MW ($)		
	2011	2011 w/o Feb & Aug	2012	2011	2011 w/o Feb & Aug	2012
$5000/MWh	60,247,604	2,945,245	1,757,030	88,156,738	1,938,250	1,554,245
$7000/MWh	104,970,127	4,380,067	2,902,841	153,256,516	3,123,633	2,905,054
$9000/MWh	149,692,650	5,814,890	4,048,651	218,356,295	4,309,015	4,255,863

In summary, the back cast for various VOLLs at each of the future SWCAPs ($5000, $7000 and $9000), using twenty-four distinct seasonal and time-of-day specific ORDCs, shows that there is a positive addition to the energy-weighted average price and the AS imbalance settlement calculation results in a net refund to the loads. As a result, the change to the total net payment to Resources also needs to be estimated to better understand the overall effect of these two results. Table 5 presents the additional net revenue to Resources taking into consideration the impacts of both the increased energy prices and Real-Time AS imbalance settlement. The negative sign for the values in the table indicates a net additional payment to the Resources under all the scenarios that were studied.

Table 2 : Change in net (energy + AS) charge to Resources for different VOLLs and minimum contingency levels (X)

VOLL	Change in Net (Energy + AS) Charge to Resources with X at 1375			Change in Net (Energy + AS) Charge to Resources with X at 1750		
	2011	2011 w/o Feb & Aug	2012	2011	2011 w/o Feb & Aug	2012
$5000/MWh	-2,263,748,410	-499,032,094	-349,087,357	-3,908,542,492	-1,046,569,996	-777,553,251
$7000/MWh	-3,637,412,917	-710,986,103	-504,279,661	-6,175,394,742	-1,487,477,702	-1,115,981,451
$9000/MWh	-5,011,077,423	-922,940,111	-659,471,967	-8,442,246,992	-1,928,385,410	-1,454,409,650

2. Real-Time Co-Optimization of Energy and Ancillary Services

Real-Time co-optimization of energy and AS will result in the appropriate valuation of energy during periods when demand is high and operating reserves are low. This valuation is accomplished through the utilization of an ORDC that results in the price of energy reflecting the opportunity cost of reserve scarcity. The current ERCOT market includes co-optimization in its organized forward DAM without an ORDC, while the Real-Time spot market does not include co-optimization of energy and AS. The Real-Time spot market only prices energy and does not include the opportunity cost of operating reserves.

The Real-Time energy and AS co-optimization proposal utilizes ORDCs which provide a mechanism of creating appropriate scarcity prices. Implementing this proposal in Real-Time will require a change to the DAM to incorporate an ORDC in order for the markets to converge. In both the DAM as well as the Real-Time spot market, maintaining power balance in the market clearing process is given the highest priority. The current ERCOT DAM is a voluntary market for buyers (demand) and sellers (supply). Demand is elastic in the DAM and thus, there is a "market based" VOLL set by the demand bids. The DAM algorithm will maintain power balance (with supply equal to demand) such that the resulting energy and AS prices reflect opportunity costs and Resources are indifferent to whether their capacity is procured for energy or for AS. In the DAM, during expected scarcity conditions, demand bids frequently set the price (at "market based" VOLL) and the resultant prices for energy and AS are high.

In the current ERCOT Real-Time spot market, demand is inelastic and energy and AS co-optimization is not performed. Resource Energy Offers or the administrative Power Balance Penalty Curve (PBPC) can set the price at or near the SWCAP during scarcity conditions. Presently, the SWCAP is the maximum price that a Resource can offer for energy. The current SWCAP was set to $4500/MWh in 2012, and will change to $5000/MWh in 2013, $7000/MWh in 2014, and $9000/MWh in 2015. If Real-Time energy and AS co-optimization is adopted, then the use of the SWCAP in Real-Time and its inter-relation with the PBPC and VOLL needs to be revisited. The design of the ORDC and the price of reserves under scarcity depend on the inter-relation between the PBPC, VOLL, ORDC and SWCAP.

There are two approaches to implementing Real-Time energy and AS co-optimization utilizing an ORDC:

<u>Approach 1:</u> If we assume that the SWCAP is the same as VOLL, then the maximum price on the PBPC would be set to SWCAP + 1. The Real-Time spot market clearing process uses the Security-Constrained Economic Dispatch (SCED) application to dispatch Resources and set prices. For each execution of SCED, the marginal offer from Resources providing reserves will be determined and the ORDC will be constructed as LOLP * (VOLL - Marginal-Offer-From-Resource-Providing-Reserves). Since the last parameter in this equation is not a fixed value and could vary for each SCED execution, the Real-Time ORDC could vary for each SCED execution as well. In this construct, the DAM will also have to be changed to calculate the marginal offer from virtual and physical Resources providing reserves and adjust the ORDC for DAM to LOLP * (VOLL - Marginal-Offer-From-virtual-or-physical-Resource-Providing-Reserves), which could possibly be different for each of the twenty-four hours studies within the DAM process. In short, this approach is needed with the current rules in order to ensure that power balance is given the highest priority. This approach, which uses a modified ORDC for each SCED execution and for each hour of the DAM, can result in a reserve price that is near zero and an energy price near SWCAP under scarcity conditions.

Approach 2: In this approach SWCAP is only applicable to the PBPC. Resources can only offer up to a new, smaller offer cap value (SWCAP_NEW). The maximum price on the PBPC will still be set to SWCAP + 1, but the ORDC will be calculated based on LOLP * (SWCAP – SWCAP_NEW). Under this approach, scarcity prices will reach SWCAP and the reserve prices will not need to be decreased under scarcity conditions as they are under Approach 1. This approach allows the ORDCs for DAM and Real-Time to be predefined for each time period of the day rather than for each SCED execution or each hour of the DAM. It also ensures that the prices for reserves are always increasing as they are depleted. In this approach, under scarcity conditions, reserve prices approach (SWCAP – SWCAP_NEW) and the price for energy approaches SWCAP.

Though these two approaches create the same Real-Time energy prices, they create different reserve prices and have different system change requirements. In addition, Approach 2 is a simpler implementation and has the effect of taking the scarcity component out of the Resource Energy Offers.

Real-Time co-optimization requires AS providers in the DAM to buy back the AS at the Real-Time price if they are not provided in Real-Time; thus, a Real-Time AS imbalance settlement structure for reserves is a part of Real-Time energy and AS co-optimization solution.

3. Interim Solution B+

"Interim Solution B+" is intended to be a close approximation of Real-Time energy and AS co-optimization. Approach 1, as described above, has been utilized in the back cast analysis performed for 2011 and 2012 with the assumption that the original marginal energy price remained unchanged. In addition, the original energy price plus the price adder is allowed to reach a maximum value of the VOLL.

Preliminary analysis of the timeframe for implementing Real-Time co-optimization of energy and AS indicated that it could not be done in the near-term. In order to provide a more gradual increase in the energy price, leading up to the SWCAP as conditions become scarce in Real-Time, two alternative approaches were proposed, "Interim Solution A" and "Interim Solution B." These approaches were filed with the PUCT on January 24, 2013 under Case 40000 [item# 369].

The "Interim Solution B" proposal removes the existing Energy Offer floor requirements from Generators for AS, and incorporates the ORDC into the determination of Real-Time prices for energy. The proposal introduces a price adder to the system wide energy price based on the ORDC which is an increasing function that values the remaining reserves as a function of the total generation in the system. While both approaches indicated above should create the desired effect of having a more gradual increase in the energy price as conditions become scarce in Real-

Time, “Interim Solution B” should provide a more accurate approximation of full Real-Time co-optimization of energy and AS and will include prices for both energy and Real-Time reserves.

During the January 24, 2014 workshop, concerns were raised about “Interim Solution B”. These concerns were focused on negative market behavior that the proposal could incentivize due to the inconsistency between the increased prices and the dispatch from the Real-Time market. These concerns included:

1. Resources ignoring dispatch instructions to “chase” the higher energy prices;
2. Entities reducing Real-Time Energy Offers to values below costs in order to offset possible inconsistencies with the DAM; and
3. Entities needing to buy back DAM energy awards in Real-Time at a higher cost due to the potential inconsistencies.

The utilization of an AS imbalance settlement was developed to address these negative incentives. The “Interim Solution B” combined with the AS imbalance settlement is what is being referred to as “Interim Solution B+.”

There are two key values that are part of “Interim Solution B+”. The first value is a price for Real-Time reserves from Load Resources providing Responsive Reserve Service (RRS) and Resources that are participating in SCED. This price serves as the price adder for the Real-Time energy price. In order to address price inconsistency between the dispatch and the final price, the remaining reserves provided by Resources minus their AS obligation are paid this price adder as well. The second value is the price calculated and used in the AS imbalance settlement for Real-Time reserves that are being provided by Offline Resources. These are Resources that are not currently available for dispatch by SECD but could be made available to SCED in 30 minutes. The AS imbalance settlement will ensure that Resources are indifferent between providing energy and reserves in Real-Time. This addresses the earlier discussed incentive concerns.

While the incentive concerns were originally raised in regards to “Interim Solution B,” it is important to recognize that similar concerns also exist with the Energy Offer floors currently in place and modified as part of “Interim Solution A.” This is specifically true for those Resources which are providing Online Non-Spinning Reserve Service (NSRS) in Real-Time that have a marginal cost lower than $120/MWh. Such a Resource has the incentive to ignore dispatch instructions in order to “chase” the higher energy price whenever the price is greater than their marginal cost. However, an AS imbalance settlement process may be less feasible under an Energy Offer floor approach due to there not being an explicit price for Real-Time reserves.

4. Methodology for Implementing Interim Solution B+

Determining the following values is a major part of implementing “Interim Solution B+:”

1. VOLL;
2. LOLP;

3. The Real-Time price for remaining reserves in the system; and
4. The AS imbalance settlements

Pending results of other studies estimating the VOLL, the back cast utilizes a range. VOLL was assumed at each of the future SWCAPs ($5000, $7000 and $9000) for the back cast. Market participant submissions and system conditions from 2011 and 2012 were utilized assuming that market behavior did not change.

The key part for back casting of "Interim Solution B+" is the determination of LOLP. LOLP depends on many factors, including the probability of forced outages, probability of load forecast error and probability of wind forecast error. It could also be different for different times of the day and for different months of the year. LOLP at a given reserve level can be interpreted as the probability of the occurrence of an event with a magnitude greater than that reserve level. A minimum contingency level (X) is chosen in order to send an appropriate scarcity price signal to maintain reliability and stability of the system. The LOLP for reserve levels below the minimum contingency level (X) will be set to one. In addition, since ERCOT is at a higher risk of shedding firm load when reserves fall near or below the minimum contingency reserve level, the LOLP curve is shifted to the right by the minimum contingency level (X) amount. The LOLP curve for a given reserve level (R) will be given as follows:

$$\pi(R) = \begin{cases} LOLP(R - X), & R - X \geq 0 \\ 1 & , R - X < 0 \end{cases}$$

LOLP is determined by analyzing historic "events," where an event is defined as the difference between the hour-ahead forecasted reserves and the reserves that were available during the Operating Hour. These events were split into twenty-four groups, comprising of four seasons and six time-of-day blocks. These groups were used to determine twenty-four distinct normal probability distributions. Seasonal and time-of-day specific curves were created to capture the potential differences between the different time periods and risk levels that occur throughout the year.

Once LOLP is determined, the next step is the calculation of the price (P_S) for reserves that are being provided by Load Resources providing RRS and Resources participating in SCED, and the price (P_{NS}) for the reserves being provided by Offline Resources not currently available to SCED but could be made available to SCED in 30 minutes. P_S and P_{NS} are functions of the LOLP at various levels of Real-Time reserves, the net value of load curtailment, and the time duration during which the reserves could be available. In this proposal, P_S and P_{NS} are determined as follows:

$$P_S = v * 0.5 * \pi_S(R_S) + P_{NS}$$
$$P_{NS} = v * (1 - 0.5) * \pi_{NS}(R_{SNS})$$

Within these formulae, v represents the net value of load curtailment and is calculated as the VOLL minus the marginal cost of energy. The marginal cost of energy is subtracted from the VOLL to ensure that the final cost of energy does not go above the SWCAP.

This approach separates the Operating Hour into two distinct time intervals, each having a length of 30 minutes (or 0.5 hours). During the first 30 minute interval only the Online reserves (R_S) are able to help prevent a loss-of-load event. In this proposal, R_S is approximated as the sum of Load Resources providing RRS and unloaded capacity up to the High Sustainable Limit (HSL) of Resources participating in SCED. For the second 30 minute period, both the Online and Offline Resources that could be made available to SCED in 30 minutes are able to help prevent a firm load shed event. In this proposal, R_{SNS} is approximated as the sum of Load Resources providing RRS, unloaded capacity up to HSL of Resources participating in SCED and Offline Resources not participating in SCED that are providing NSRS or have a cold start time less than or equal to 30 minutes.

Separate LOLP curves (π_S & π_{NS}) are determined for these two distinct time intervals within the hour by using the historically observed errors in the estimated reserves based on season and time-of-day block. For each SCED interval, the price adder for energy is then determined using the LOLP curves (π_S & π_{NS}), Online Reserves (R_S), Offline Reserves (R_{NS}), VOLL and the current marginal cost of energy. The average price adder for a given year is then calculated as the energy-weighted average of the SCED interval price adders in the year.

The AS imbalance is calculated for each QSE by comparing the net AS Supply Responsibility of the QSE going into the hour and the net AS available from the QSE in Real-Time. The AS Supply Responsibility of the QSE is based on the QSE's Self-Scheduled AS, DAM AS awards, net AS trade, AS failures and replacements and Supplemental Ancillary Service Market (SASM) awards. If the QSE is short on AS in Real-Time, then they will be charged the price adder for the short amount and if the QSE is long on AS in Real-Time, then they will be paid the price adder for the long amount.

5. Detailed Results

The back cast for various VOLLs at each of the future SWCAPs ($5000, $7000 and $9000) and using the twenty-four distinct seasonal and time-of-day specific ORDCs, shows that there is a positive addition to the energy-weighted average price. An energy-weighted average price of $3.45/MWh occurs in 2012 with a VOLL of $7000/MWh and a minimum contingency level of 1750MW. In addition, the back cast also shows a $2.9M refund to loads from the AS imbalance settlement and $1.12B in additional payments for energy. The potential increase in PNM is $24,809/MW. Increasing the SWCAP from $3000/MWh to $7000/MWh and not applying interim solution B+, would yield a PNM increase of $5,883/MW. In short, the back cast results show that the market impacts of "Interim Solution B+" depends on the parameters for the ORDC.

Table 6 provides the summary of P_S for different values of VOLL and minimum contingency levels (X).

Table 3 : Energy-weighted average energy price adder (and Online reserve price) P_S ($/MWh) for 2011 & 2012 with different VOLLs and minimum contingency levels (X)

VOLL	Energy-weighted average Ps with X at 1375 ($/MWh)			Energy-weighted average Ps with X at 1750 ($/MWh)		
	2011	2012	2011 & 2012 combined	2011	2012	2011 & 2012 combined
$5000/MWh	7.00	1.08	4.08	12.03	2.40	7.28
$7000/MWh	11.27	1.56	6.48	19.06	3.45	11.35
$9000/MWh	15.54	2.05	8.87	26.08	4.50	15.42

Table 7 provides the summary of the Offline reserve price (P_{NS}) for different values of VOLL and minimum contingency levels (X).

Table 4 : Energy-weighted average price of Offline reserves P_{NS} ($/MWh) for 2011 & 2012 with different VOLLs and minimum contingency levels (X)

VOLL	Energy-weighted average P_{NS} with X at 1375($/MWh)			Energy-weighted average P_{NS} with X at 1750 ($/MWh)		
	2011	2012	2011 & 2012 combined	2011	2012	2011 & 2012 combined
$5000/MWh	3.84	0.48	2.18	6.08	0.92	3.53
$7000/MWh	6.15	0.69	3.45	9.63	1.33	5.53
$9000/MWh	8.46	0.91	4.73	13.18	1.73	7.53

Due to the increase in energy prices resulting from the proposal, the potential impacts on PNM were also analyzed. Table 8 shows the additional PNM from the approach being presented. In addition, a study was also performed to determine what the potential PNM impacts to 2011 and 2012 may have been if the Real-Time market simply had a higher SWCAP during those study years. Table 9 shows what the additional PNM would have been by solely increasing the SWCAP to the various values of VOLL being evaluated as part of the back cast. The actual PNM for 2011 and 2012 was $125,001/MW and $33,952/MW, respectively.

Table 5 : Estimated additional PNM (in $/MW) from "Interim Solution B+" for 2011 & 2012 with different VOLLs and minimum contingency levels (X)

VOLL	Total Additional PNM under Interim Solution B+ with X at 1375 ($/MW)		Total Additional PNM under Interim Solution B+ with X at 1750 ($/MW)	
	2011	2012	2011	2012
$5000/MWh	38,544	7,740	67,892	17,267
$7000/MWh	62,141	11,189	107,327	24,809
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Table 6 : Estimated additional PNM ($/MW) for 2011 and 2012 by only increasing the SWCAP

SWCAP	Total Additional PNM if SWCAP Increased to VOLL ($/MW)	
	2011	2012
$5000/MWh	57,631	2,877
$7000/MWh	114,168	5,883
$9000/MWh	170,706	8,889

As part of the "Interim Solution B+" proposal, a Real-Time AS imbalance settlement is also introduced. This is intended to account for the fact that Resources may have a different amount of reserves available in Real-Time relative to the amount that they were obligated to provide based on activities in the Day-Ahead Market (DAM) and Adjustment Period. This can result in QSEs needing to purchase reserves in Real-Time to cover those obligations. Due to the different prices for Online and Offline reserves, the AS imbalance settlement analysis is split up to look at each of the two reserve categories individually. Table 10 and Table 11 present the AS imbalance settlement subdivided into Online and Offline imbalance settlements for 2011 & 2012 with different VOLL and minimum contingency levels (X). Table 12 then presents the net of the Online and Offline AS imbalance settlements taking all types of reserves into consideration. A positive sign for the values in these three tables represent a charge to Resources and it can be seen in Table 12 that the net result is a refund to the loads for the AS imbalance settlement.

Table 7 : Net AS imbalance settlement ($) charge to Resources for 2011 & 2012 with different VOLLs and a minimum contingency level (X) of 1375 MW

VOLL	Net AS Imbalance Settlement for Online Reserves ($)			Net AS Imbalance Settlement for Offline Reserves ($)		
	2011	2011 w/o Feb & Aug	2012	2011	2011 w/o Feb & Aug	2012
$5000/MWh	60,770,786	3,314,157	2,094,851	-523,182	-368,912	-337,821
$7000/MWh	105,722,935	4,902,316	3,381,553	-752,808	-522,249	-478,712
$9000/MWh	150,675,084	6,490,475	4,668,255	-982,434	-675,585	-619,604

Table 8 : Net AS imbalance settlement ($) charge to Resources for 2011 & 2012 with different VOLLs and a minimum contingency level (X) of 1750 MW

VOLL	Net AS Imbalance Settlement for Online Reserves ($)			Net AS Imbalance Settlement for Offline Reserves ($)		
	2011	2011 w/o Feb & Aug	2012	2011	2011 w/o Feb & Aug	2012
$5000/MWh	89,248,751	2,745,065	2,329,604	-1,092,013	-806,815	-775,359
$7000/MWh	154,818,072	4,263,919	4,000,690	-1,561,556	-1,140,286	-1,095,636
$9000/MWh	220,387,394	5,782,772	5,671,775	-2,031,099	-1,473,757	-1,415,912

Table 9 : Net AS imbalance settlement ($) charge to Resources for 2011 & 2012 with different VOLLs and minimum contingency levels (X)

VOLL	Net AS Imbalance Settlement for All Reserves with X at 1375MW ($)			Net AS Imbalance Settlement for All Reserves with X at 1750MW ($)		
	2011	2011 w/o Feb & Aug	2012	2011	2011 w/o Feb & Aug	2012
$5000/MWh	60,247,604	2,945,245	1,757,030	88,156,738	1,938,250	1,554,245
$7000/MWh	104,970,127	4,380,067	2,902,841	153,256,516	3,123,633	2,905,054
$9000/MWh	149,692,650	5,814,890	4,048,651	218,356,295	4,309,015	4,255,863

In summary, the back cast for various VOLLs at each of the future SWCAPs ($5000, $7000 and $9000), using twenty-four distinct seasonal and time-of-day specific ORDCs, shows that there is a positive addition to the energy-weighted average price and the AS imbalance settlement calculation resulting in a net refund to the loads. As a result, the change to the total net payment to Resources also needs to be estimated to better understand the overall effect of these two results. Table 13 presents the additional net revenue to Resources taking into consideration the impacts of both the increased energy prices and Real-Time AS imbalance settlement. The negative sign for the values in the table indicates a net additional payment to the Resources under all the scenarios that were studied as part of the back cast.

Table 10 : Change in Net (energy + AS) charge to Resources with minimum contingency levels of 1375 MW and 1750 MW

VOLL	Change in Net (Energy + AS) Charge to Resources with X at 1375			Change in Net (Energy + AS) Charge to Resources with X at 1750		
	2011	2011 w/o Feb & Aug	2012	2011	2011 w/o Feb & Aug	2012
$5000/MWh	-2,263,748,410	-499,032,094	-349,087,357	-3,908,542,492	-1,046,569,996	-777,553,251
$7000/MWh	-3,637,412,917	-710,986,103	-504,279,661	-6,175,394,742	-1,487,477,702	-1,115,981,451
$9000/MWh	-5,011,077,423	-922,940,111	-659,471,967	-8,442,246,992	-1,928,385,410	-1,454,409,650

6. Appendix

The following sections provide additional detail to the methodology used in back casting the “Interim Solution B+” proposal and provide the derivation of how the proposal approximates Real-Time co-optimization of energy and AS.

6.1. Appendix I: Detailed Methodology for Back Cast

Determining the following values is a major part of implementing “Interim Solution B+:”

1. VOLL;
2. LOLP;
3. The price for remaining reserves in the system; and
4. The AS imbalance settlements.

For back casting, VOLL is assumed to be at each of the future values of SWCAP ($5000, $7000 and $9000).

6.1.1. Determining LOLP

For back casting, LOLP is determined by analyzing historic events defined as the difference between the hour-ahead forecasted reserves with the reserves that were available in Real-Time during the Operating Hour. These events were split into twenty-four groups, comprising of four seasons and six time-of-day blocks per day. These groups were used to determine twenty-four distinct normal probability distributions of the events which will determine the LOLP for the corresponding season and time block. The detail logic used for determining LOLP is described as below:

1) For each Operating Hour in the study period, calculate the system-wide Hour-Ahead (HA) reserve using the snapshot of last HRUC for the Operating Hour (at the end of Adjustment Period):

$$HA\ Reserve = RUC\ Online\ COP\ HSL - (RUC\ Load\ Forecast + RUC\ DCTIE\ Load) + RUC\ COP\ OFFNS\ Schedule$$

2) For each SCED interval in the study period, calculate the system-wide available SCED reserve using SCED telemetry and solution as:

$$SCED\ Reserve = SCED\ Online\ HSL - SCED\ BP + SCED\ OFFNS\ Schedule$$

3) For each Operating Hour in the study period, calculate the hourly average system-wide SCED reserve by averaging the interval SCED reserve in step 2).

4) For each Operating Hour in the study period, calculate the system wide Reserve Error as:

$$Reserve\ Error = HA\ Reserve - SCED\ Reserve\ (Hourly\ Average)$$

5) For each Operating Hour in the study period, allocate it to the corresponding season and time block. So all the hours will be split into 24 distribution groups developed for the analysis based on the Season and the time of day:

   - 4 Seasons of Winter, Spring, Summer and Fall
   - 6 time-of-day blocks each consisting of 4 hours

6) Calculate the mean (μ) and standard deviation (σ) for each of the twenty-four distinct LOLP distributions using the calculated Reserve Error in step 4). The detail results are illustrated in Table 11. This hourly error is normally distributed and hence $LOLP$ for a given value y can be calculated:

$$LOLP(\mu, \sigma, y) = 1 - CDF(\mu, \sigma, y)$$

   Where CDF is the Cumulative Distribution Function of the normal distribution with mean μ and standard deviation σ.

Table 11 : LOLP distributions by season and time-of-day block

Season	For Hours	μ	σ
Winter (Month 12, 1, 2)	1-2 and 23-24	185.14	1217.89
	3-6	76.28	1253.93
	7-10	136.32	1434.64
	11-14	-218.26	1441.00
	15-18	-53.67	1349.52
	19-22	-183.00	1129.31
Spring (Month 3,4,5)	1-2 and 23-24	245.76	1174.61
	3-6	460.41	1313.46
	7-10	348.16	1292.36
	11-14	-491.91	1332.05
	15-18	-253.77	1382.60
	19-22	-436.09	1280.47
Summer (Month 6,7,8)	1-2 and 23-24	374.88	1503.97
	3-6	1044.81	1252.25
	7-10	339.01	1679.70
	11-14	-695.94	1251.05
	15-18	-270.54	1284.96
	19-22	-730.33	1331.49
Fall (Month 9, 10,11)	1-2 and 23-24	15.90	1044.88
	3-6	478.97	1014.02
	7-10	322.65	1036.07
	11-14	-473.16	1293.83
	15-18	-422.21	1246.49
	19-22	-177.76	1231.14

6.1.1.1. Calculation of R_S and R_{SNS}

R_S is the reserves from Resources participating in SCED plus the RRS from Load Resources. R_{SNS} is equal to R_S plus the reserves from Resources that are not currently available to SCED but could be made available in 30 minutes.

1) R_S is calculated based on SCED telemetry and solution as:

$$R_S = (1 - DF) * (HSL - HSL_{WGR} - HSL_{NUC}) - (BP - BP_{WGR} - BP_{NUC}) + RRS_{load}$$

Where

- DF is the discount applied to the real-time HSLs of Generators. For this analysis, a DF of 0.01 or 1% is assumed.
- HSL and BP are the system total SCED online HSL and base point respectively.
- HSL_{WGR} and HSL_{NUC} are the system total SCED online HSL of wind and nuclear Resources respectively.
- BP_{WGR} and BP_{NUC} are the system total SCED Online base point of wind and nuclear Resources respectively.
- RRS_{load} is the system total SCED RRS schedules from Load Resources.

2) R_{SNS} is calculated based on SCED telemetry and solution as

$$R_{SNS} = R_S + (1 - DF) * (HSL_{OFFNS} + HSL_{OFF30})$$

Where

- HSL_{OFFNS} is the system total HSL of Offline Generators providing Non-spin
- HSL_{OFF30} is the system total HSL of Offline and available Generators that can be started from a cold temperature state in 30 minutes

6.1.1.2. Calculation of $\pi_S(R_S)$ and $\pi_{NS}(R_{SNS})$

$\pi_S(R_S)$ and $\pi_{NS}(R_{SNS})$ are functions that describe the Loss of Load Probability (LOLP) at various reserve levels.

1) Calculation of $\pi_S(R_S)$:

$\pi_S(R_S)$ is a function of the Real-Time reserves that should be available in the first 30 minutes of the hour and is intended to capture the LOLP for that level of reserves. The general equation for $\pi_S(R_S)$ is:

$$\pi_S(R_S) = \begin{cases} LOLP_S(R_S - X), & R_S - X \geq 0 \\ 1 & , R_S - X < 0 \end{cases}$$

Where

- X in this equation is a minimum contingency level and represents a level of reserves at which ERCOT may need to begin to shed firm load.

- $LOLP_S$ is the $LOLP$ function for the spinning reserve.

$LOLP_S$ is different from the 60 minutes $LOLP$ in Table 11 which is calculated based on the hourly error analysis. The reserves are classified into two categories; those that are being provided by Resources in SCED and Load Resources providing RRS and those that are being providing by Resources that are not currently available to SCED but could be made available within 30 minutes. Since the first reserve type is available immediately, those reserves are the only ones considered to be available to respond to any event that happens in the first 30 minutes of the hour. All reserve types are then considered to be available to respond to events that happen in the second 30 minutes of the hour. From the hourly error analysis, a mean (μ) and standard deviation (σ) for the 60 minute $LOLP$ are determined for each of the different seasons and time blocks. Because the error analysis is hourly, to capture the events within the first 30 minutes for $\pi_S(R_S)$, the μ and σ needs to be scaled to reflect the 30 minute timeframe, with $\delta = 0.5$ hours :

$$\mu' = \delta * \mu = 0.5\mu$$
$$\sigma' = \frac{\delta}{\sqrt{\delta^2 + (1-\delta)^2}} * \sigma = 0.707\sigma$$

So the $LOLP_S$ can be calculated based on the 60 minute $LOLP$ as follows:

$$LOLP_S(\mu', \sigma', y) = LOLP(0.5\mu, 0.707\sigma, y) = 1 - CDF(0.5\mu, 0.707\sigma, y)$$

For simplification and ease of implementation, a piecewise linear approximation is used for the nonlinear curve $\pi_S(R_S)$ as given below:

- For R_S between 0 and X, set $\pi_S(R_S)$ equal to 1
- For $R_S = Reg_{Up} + RRS_{Load}$, set $\pi_S(R_S)$ equal to $LOLP_S\left(Reg_{Up} + RRS_{Load} - X\right)$
- For $R_S = Reg_{Up} + RRS_{All}$, set $\pi_S(R_S)$ equal to $LOLP_S\left(Reg_{Up} + RRS_{All} - X\right)$
- For $R_S = Reg_{Up} + RRS_{All} + NonSpin$, set $\pi_S(R_S)$ equal to $LOLP_S\left(Reg_{Up} + RRS_{All} + NonSpin - X\right)$
- Other breakpoints for R_S as the LOLP approaches zero
- Linearly interpolate the values between these points

The breakpoints used in this analysis are X, 1900, 3300, 4800, 6000 and 8000 MW. 1375 and 1750 MW are analyzed as potential values of X. 24 $\pi_S(R_S)$ curves are developed for the analysis based on the season and the time of day. One example of the $\pi_S(R_S)$ curve is shown in Figure 1.

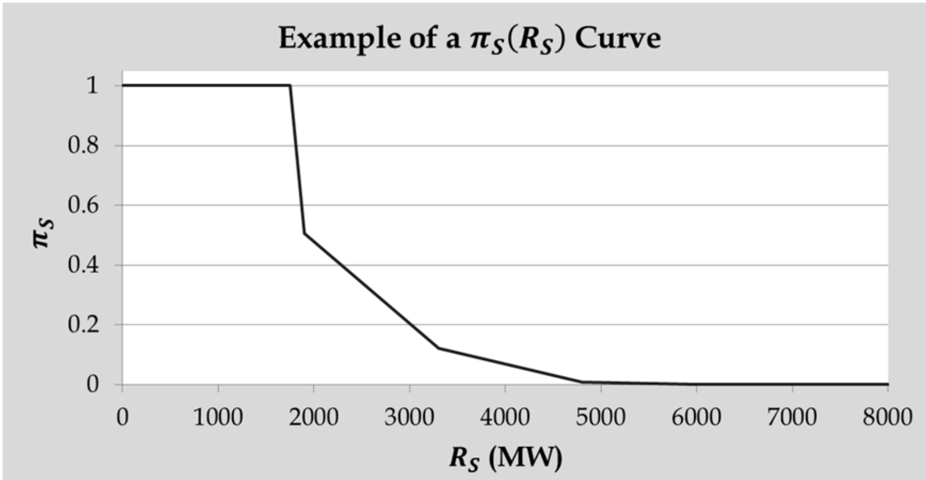

Figure 1

2) Calculation of $\pi_{NS}(R_{\text{SNS}})$:

$\pi_{NS}(R_{\text{SNS}})$ is a function of all the Real-Time reserves that can be expected to be available with the hour and is intended to capture the LOLP for that level of reserves based on events that happen in an hour. The general equation for $\pi_{NS}(R_{\text{SNS}})$ is:

$$\pi_{NS}(R_{\text{SNS}}) = \begin{cases} LOLP\ (R_{\text{SNS}} - X), R_{\text{SNS}} - X \geq 0 \\ 1 \qquad\qquad\quad , R_{\text{SNS}} - X < 0 \end{cases}$$

This is similar to $\pi_S(R_S)$ but the key differences here are the types of reserves considered and the μ and σ that are used in calculating LOLP for the various breakpoints

- The total online and offline applies for the full change in net load over the hour and there is no scaling adjustments needed for μ and σ in the $\pi_{NS}(R_{\text{SNS}})$ calculations
- Again, X in this equation is a minimum contingency level

Like $\pi_S(R_S)$, twenty-four individual piecewise linear approximations are created for $\pi_{NS}(R_{\text{SNS}})$ using the same MW breakpoints.

6.1.2. Determination of Price Adder

Once LOLP is determined, the next step is the calculation of the price P_S for reserves that are being provided by Load Resources providing Responsive Reserve and Generators participating in SCED and the price P_{NS} for the reserves being provided by offline Generators not currently available to SCED but could be made available to SCED in 30 minutes. P_S and P_{NS} are functions of the LOLP at various levels of Real-Time reserves, the net value of load curtailment, and time duration during which the reserves are available. In this proposal, P_S and P_{NS} are determined as follows:

$$P_S = v * 0.5 * \pi_S(R_S) + P_{NS}$$
$$P_{NS} = v * (1 - 0.5) * \pi_{NS}(R_{\text{SNS}})$$

$$v = VOLL - Marginal\ Offer$$

Where v represents the net value of load curtailment and is calculated as the VOLL minus the marginal cost of energy. Marginal cost of energy is subtracted from VOLL to reflect the scarcity value of the marginal dispatch capacity and to ensure that the final cost of energy does not go above the SWCAP.

As discussed in previous section, R_S and R_{SNS} can be calculated for each SCED interval. Each SCED interval can also be mapped to one of the 24 $\pi_S(R_S)$ and $\pi_{NS}(R_{\text{SNS}})$ curves respectively. So the $\pi_S(R_S)$ and $\pi_{NS}(R_{\text{SNS}})$ can be calculated using the interpolation on the curve. Let us use $\pi_S(R_S)$ as an example. The same logic can be applied to the calculation of $\pi_{NS}(R_{SNS})$. For (P_i, Q_i), the breakpoint i on the $\pi_S(R_S)$ curve (P_i is the π_S probability value and $Q_i$is the R_S MW value) the logic can be illustrated as follows:

- Determine the segment of the piecewise linear $\pi_S(R_S)$ curve in which R_S will fall : assume $Q_i \leq R_S < Q_{i+1}$ then R_S is between break point i and $i + 1$
- Calculate the slope for this segment as
$$slope = \frac{P_{i+1} - P_i}{Q_{i+1} - Q_i}$$
- Calculate $\pi_S(R_S)$ as
$$\pi_S(R_S) = slope * (R_S - Q_i) + P_i$$

Once $\pi_S(R_S)$ and $\pi_{NS}(R_{SNS})$ are calculated, P_S and P_{NS} can be calculated for each SCED interval using the formulation at the beginning of this section. The energy-weighted average P_S and P_{NS} can be calculated based on all the SCED intervals in the study period:

$$Average\ P_S = \frac{\sum(P_S * BP * SCED\ Length)}{\sum(BP * SCED\ Length)}$$
$$Average\ P_{NS} = \frac{\sum(P_{NS} * BP * SCED\ Length)}{\sum(BP * SCED\ Length)}$$

For this equation, “SCED Length” is equal to the duration of the SCED interval in hours.

6.1.3. Determining Ancillary Service Imbalance Payment

Once the prices for the reserves are calculated the AS imbalance is calculated for each QSE by determining the net AS Supply Responsibility of the QSE going into the hour and the net AS available from the QSE in Real-Time. The AS Supply Responsibility of the QSE is based on QSEs Self-Schedule AS, DAM AS awards, net AS trade, AS failures and replacements and SASM awards. If the QSE is short on AS in Real-Time then they will be charged the price adder for the short amount and if the QSE is long on AS in Real-Time then they will be paid the price adder for the long amount.

The AS Responsibility for each AS type (Reg-Up/RRS/Non-Spin) for each hour for each QSE can be calculated as follows:

$$
\begin{aligned}
&AS\ Responsibility\\
&\qquad = Self\ Scheduled\ AS + AS\ Trade\ Sold + DAM\ AS\ Award\\
&\qquad + SASM\ AS\ Award - AS\ Trade\ Bought - SASM\ AS\ Replacement\\
&\qquad - SASM\ AS\ Failure\ to\ Provide
\end{aligned}
$$

In this study, the Hour-ahead (HA) AS Responsibility is used as the final AS Responsibility for the QSE, i.e. AS Responsibility at the end of Adjustment Period. The Hour-ahead Online reserve is calculated as the sum of Reg-Up, RRS and Online Non-Spin:

$$R_S\_HA = REGUP\_HA + RRS\_HA + NSPIN_{online}\_HA$$

Since the Non-spin responsibility doesn't differentiate Online and Offline Non-spin, the Hour-ahead Offline Non-spin can be assumed the same as Real-Time Offline Non-spin. So the Hour-ahead Online Non-Spin can be calculated as:

$$NSPIN_{online}\_HA = NSPIN\_HA - OFFNS\_RT$$

The Real-Time Online reserve imbalance in MW for each SCED interval for each QSE can be calculated as:

$$RT\ R_S\ Imbalance = R_S - R_S\_HA$$

The Real-Time Offline reserve imbalance in MW for each SCED interval for each QSE can be calculated as:

$$RT\ R_{NS}\ Imbalance = HSL_{OFF30}$$

The payment or charge in dollars for the AS imbalance for each SCED interval for each QSE is then calculated as:

$$
\begin{aligned}
RT\ Online\ Reserve\ Imbalance\ Amount &= (-1) * P_S * RT\ R_S\ Imbalance * SCED\ Length\\
&= (-1) * P_S * (R_S - R_S\_HA) * SCED\ Length
\end{aligned}
$$

$$
\begin{aligned}
RT\ Offline\ Reserve\ Imbalance\ Amount &= (-1) * P_{NS} * RT\ R_{NS}\ Imbalance * SCED\ Length\\
&= (-1) * P_{NS} * HSL_{OFF30} * SCED\ Length
\end{aligned}
$$

For the dollar amount, a negative value indicates an ERCOT payment to a QSE and a positive value indicates an ERCOT charge to the QSE.

The system total payment or charge for AS imbalance for each SCED interval is the sum of the QSE specific AS imbalance amounts for all the QSEs for the particular SCED interval. Since P_S and P_{NS} are at the system level, the QSE specific AS imbalance formulation will hold true for the system total AS imbalance, except the R_S, R_S_HA and HSL_{OFF30} will be sum up to the system level.

In addition, the energy payment for the price adder P_S for each QSE for each SCED interval can be calculated as:

$$Energy\ Payment = (-1) * P_S * BP * SCED\ Length$$

The net payment to the QSE including both the energy payment and AS imbalance amount (online and offline) can be calculated as:

$$Net\ Payment = Energy\ Payment + \ RT\ Online\ Reserve\ Imbalance\ Amount \\ + \ RT\ Offline\ Reserve\ Imbalance\ Amount$$

The system total net payment can be summed across all the QSEs using the equation above.

6.2. Appendix II: Interim Solution B + Theory

6.2.1. An Approximation Foundation for an ORDC

This section summarizes a series of steps to approximate the full Real-Time energy and AS co-optimization ORDC and be explicit about the inclusion of the costs of generation and reserves to produce the implied scarcity price. Here the focus is on responsive reserves. The various variables and functions include:

d : Vector of locational demands

g_R : Vector of locational responsive generation

r_R : Vector of locational responsive reserves

g_{NR} : Vector of locational generation not providing reserves

$B(d)$: Benefit function for demand

$C_k(g_k)$: Cost function for generation offers

K_k : Generation Capacity

$f(x)$: Probability for net load change equal to x

H, b: Transmission Constraint Parameters

i : Vector of ones.

Assume that unit commitment is determined. The stylized economic dispatch model includes an explicit description of the expected value of the use of reserves. This reserve description allows for a one dimensional change in aggregate net load, x, and an asymmetric response where positive net load changes are costly and met with reserves and negative changes in net load are ignored. This model is too difficult to implement but it provides an interpretation of a set of assumptions that leads to an approximate ORDC. Here we ignore minimum reserve requirements to focus on the expected cost of the reserve dispatch.

The central formulation treats net load change x and use of reserve, δ_x , to avoid involuntary curtailment. This produces a benefit minus cost of $VOLL \cdot \left(i^t \delta_x\right) - \left(C_R\left(g_R + \delta_x\right) - C_R\left(g_R\right)\right)$ and this is weighted by the probability $f(x)$. This term enters the objective function summed for all non-negative values of x. The basic formulation includes:

$$\underset{d,g_R,g_{NR},r_R,\delta_x\geq 0;y}{Max} \quad B(d)-C_R(g_R)-C_{NR}(g_{NR})+\sum_{x\geq 0}\left(VOLLi^t\delta_x-\left(C_R(g_R+\delta_x)-C_R(g_R)\right)\right)f(x)$$

(1)

$$\begin{array}{rcc}
d-g_R-g_{NR}=y & \text{Net Loads} & \rho \\
i^t y=0 & \text{Load Balance} & \lambda \\
Hy\leq b & \text{Transmission Limits} & \mu \\
g_R+r_R\leq K_R & \text{Responsive Capacity} & \theta_R \\
i^t\delta_x\leq x,\ \forall x & \text{Responsive Utilization} & \gamma_x \\
\delta_x\leq r_R,\ \forall x & \text{Responsive Limit} & \varphi_x \\
g_{NR}\leq K_{NR} & \text{Generation Only Capacity} & \theta_{NR}.
\end{array}$$

This model accounts for all the uncertain net load changes weighted by the probability of outcome and allows for the optimal utilization of reserve dispatch in each instance. This problem could produce scarcity prices that could differ across locations.

To approach the assessment of how to approximate reserves with a common scarcity price across the system, we need to further simplify this basic problem as follows:

1. Treat the utilization of reserves as a one-dimensional aggregate variable.
2. Replace the responsive reserve limit vector with a corresponding aggregate constraint on total reserves.
3. Utilize an approximation of the cost function, $\hat{C}$, for the aggregate utilization of reserves, and further approximate the change in costs with the derivative of cost times the utilization of reserves.

This set of assumptions produces a representation for the use of a single aggregate level of reserves for the system:

$$\underset{d,g_R,g_{NR},r_R,\delta_x\geq 0;y}{Max} \quad B(d)-C_R(g_R)-C_{NR}(g_{NR})+\sum_{x\geq 0}\left(VOLL\delta_x-\partial\hat{C}_R\left(i^t g_R\right)\delta_x\right)f(x)$$

(2)

$$\begin{array}{rcc}
d-g_R-g_{NR}=y & \text{Net Loads} & \rho \\
i^t y=0 & \text{Load Balance} & \lambda \\
Hy\leq b & \text{Transmission Limits} & \mu \\
g_R+r_R\leq K_R & \text{Responsive Capacity} & \theta_R \\
\delta_x\leq x,\ \forall x & \text{Responsive Utilization} & \gamma_x \\
\delta_x\leq i^t r_R,\ \forall x & \text{Responsive Limit} & \varphi_x \\
0\leq r_R, & \text{Explicit Sign Constraint} & \omega_R \\
g_{NR}\leq K_{NR} & \text{Generation Only Capacity} & \theta_{NR}.
\end{array}$$

This formulation provides a reasonably transparent interpretation of the implied prices. Focusing on an interior solution for all the variables except r_R, we would have locational prices related to the marginal benefits of load:

$$(3) \qquad \rho = \nabla B(d).$$

The same locational prices connect to the system lambda and the cost of congestion for the binding transmission constraints.

$$(4) \qquad \rho = \lambda i + H^t \mu.$$

The locational prices equate with the marginal cost of generation-only plus the cost of scarcity when this generation is at capacity, which appears in the usual form.

$$(5) \qquad \rho = \nabla C_{NR}\left(g_{NR}\right) + \theta_{NR}.$$

The locational prices equate with the marginal cost of responsive generation and display the impact of reserve scarcity. First, the impact of changing the base dispatch of responsive generation implies:

$$\rho = \nabla C_R\left(g_R\right) + \sum_{x \geq 0}\left(\partial^2 \hat{C}_R\left(i^t g_R\right)\delta_x i\right) f\left(x\right) + \theta_R.$$

The second order term captures the effect of the base dispatch of responsive dispatch on the expected cost of meeting the reserve utilization. This term is likely to be small. For example, if we assume that the derivative $\partial \hat{C}_R$ is constant, then the second order term is zero.

When we account for the base dispatch of reserves, we have:

$$\theta_R = \sum_{x \geq 0} \varphi_x i + \omega_R .$$

When accounting for utilization of the reserves, we have:

$$\gamma_x + \varphi_x = \left(VOLL - \partial \hat{C}_R\left(i^t g_R\right)\right) f\left(x\right).$$

Let $r = i^t r_R$. Then for $x \leq r, \quad \varphi_x = 0; \quad x \geq r, \quad \gamma_x = 0$. Hence,

$$\theta_R = \sum_{x \geq r} \varphi_x i + \omega_R = \left(VOLL - \partial \hat{C}_R\left(i^t g_R\right)\right)\left(1 - F\left(r\right)\right) i + \omega_R .$$

Combining these, we can rewrite the locational price as:

$$(6) \qquad \rho = \nabla C_R\left(g_R\right) + \sum_{x \geq 0}\left(\partial^2 \hat{C}_R\left(i^t g_R\right) i \delta_x\right) f\left(x\right) + \left(VOLL - \partial \hat{C}_R\left(i^t g_R\right)\right)\left(1 - F\left(r\right)\right) i + \omega_R.$$

Equations (3) thru (6) capture our approximating model for aggregate responsive reserves. Here $1 - F\left(r\right) = LOLP\left(r\right)$. The term $\left(VOLL - \partial \hat{C}_R\left(i^t g_R\right)\right)\left(1 - F\left(r\right)\right)$ in (6) is the scarcity price of the ORDC. If the second order terms in (6) are dropped, then the scarcity price is the only change from the conventional generation only model. In practice, we would have to update this model to account for minimum reserve levels, non-spin, and so on, but these changes would be the same as the discussion where we included an estimate of $\overline{c} \approx \partial \hat{C}_R$ in defining the net value of operating reserves $v \approx VOLL - \overline{c}$.

Note that under these assumptions the scarcity price is set according to the opportunity cost using $\hat{C}$ for the marginal responsive Generator in the base dispatch. Depending on the accuracy of the estimate in $\hat{C}$, this seeks to maintain that the energy price plus scarcity price never exceeds the value of lost load.

Providing a reasonable estimate for $\hat{C}$ could be done either as an (i) exogenous constant, (ii) through a two pass procedure, or (iii) approximately in the dispatch. For example, a possible procedure would define the approximating cost function as the least unconstrained cost,

$$\hat{C}\left(\hat{g}_R\right) = Min\left\{C\left(g_R\right) \middle| \hat{g}_R = i^t g_R\right\}.$$

This information would be easy to evaluate before the dispatch.

The purpose of models (1) and (2) above is not to design an implementation. The purpose is to illustrate a set of assumptions that would produce a simplified ORDC and how to select the parameters of the model

6.2.2. ORDC for Multiple Reserves

The ERCOT practice distinguishes several types of reserves. Setting aside regulation, the principal distinction is between "responsive" reserves (R) and "non-spin" reserves (NS). The ORDC framework can be adapted to include multiple reserves. This section summarizes one such modeling approach and relates it to the co-optimization examples above. The main distinction is that "responsive" reserves are spinning and have a quick reaction time. These reserves would be available almost immediately and could provide energy to meet increases in net load over the whole of the operating reserve period. By comparison, non-spin reserves are slower to respond and would not be available for the entire period.

The proposed model of operating reserves approximates the complex dynamics by assuming that the uncertainty about the unpredicted change in net load is revealed after the basic dispatch is determined. The probability distribution of change in net load is interpreted as applying the change over the uncertain reserve period, say the next hour, divided into two intervals. Over the first interval, of duration (δ), only the responsive reserves can avoid curtailments. Over the second interval of duration (1- δ), both the responsive and non-spin reserves can avoid involuntary load shedding.

This formulation produces different values for the responsive and non-spin reserves. Let v be the net value of load curtailment, defined as the value of lost load less the avoided cost of energy dispatch offer for the marginal reserve. The interpretation of the prices of reserves, P_R and P_{NS}, is the marginal impact on the load curtailment times $Lolp$, the probability of the net change in load being greater that the level of reserves, r_R and r_{NS}. This marginal value differs for the two intervals, as shown in the following table:

Marginal Reserve Values		
	Interval I	Interval II
Duration	δ	1-δ
P_R	$vLolp\left(r_R\right)$	$vLolp\left(r_R + r_{NS}\right)$
P_{NS}	0	$vLolp\left(r_R + r_{NS}\right)$

This formulation lends itself to the interpretation of Figure 2 where there are two periods with different demand curves and the models are nested. In other words, responsive reserves r_R can meet the needs in both intervals and the non-spin reserves r_{NS} can only meet the needs for the second interval.

The resulting prices satisfy:

$$\begin{aligned} &P_R = v*\left(\delta * Lolp\left(r_R\right) + \left(1-\delta\right)* Lolp\left(r_R + r_{NS}\right)\right) = v*\delta * Lolp\left(r_R\right) + P_{NS}, \\ &P_{NS} = v*\left(1-\delta\right)* Lolp\left(r_R + r_{NS}\right). \end{aligned} \tag{7}$$

This formulation lends itself to a relatively easy implementation in the co-optimization model.

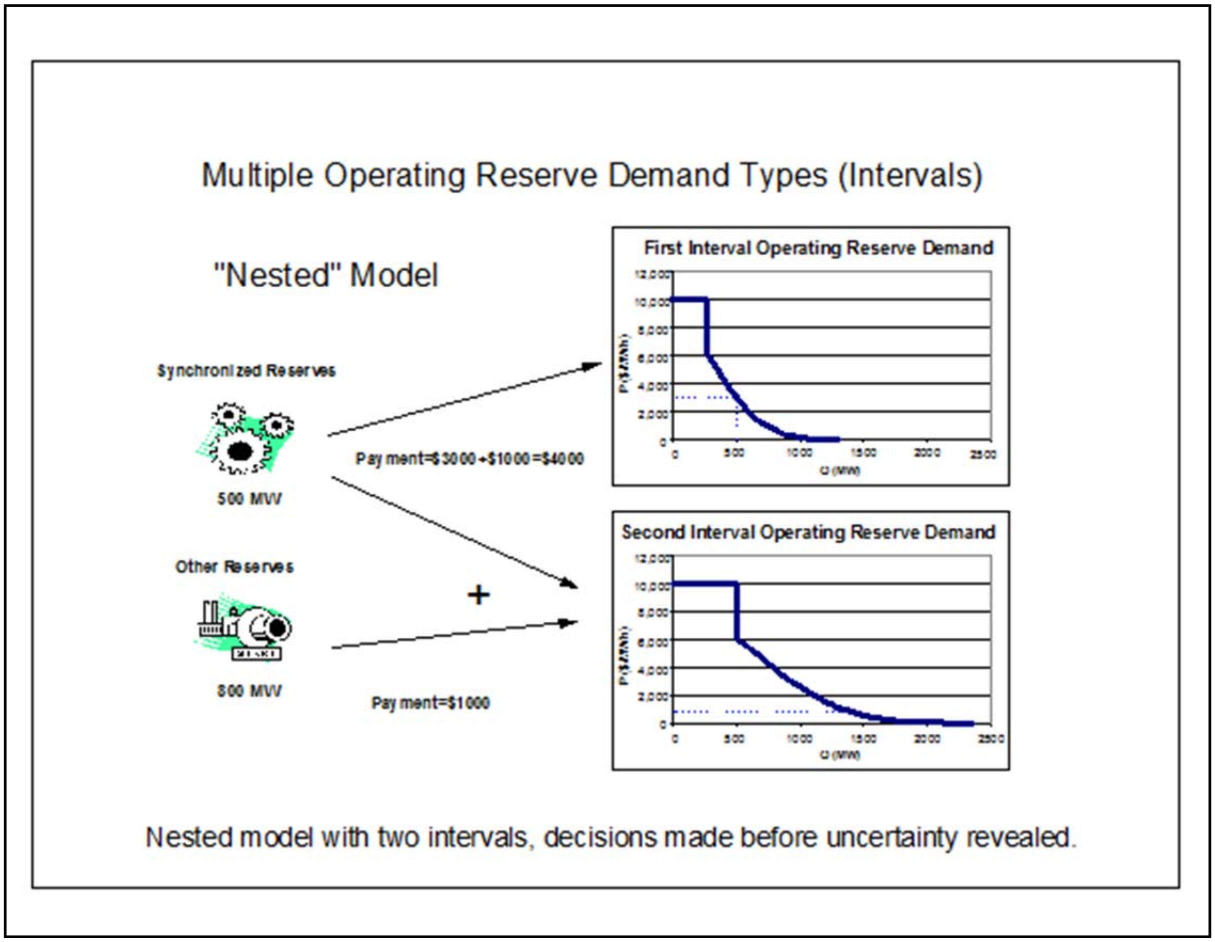

Figure 2

As shown later, the introduction of multiple types of ORDCs does not much affect the economic dispatch model for real time. The same properties apply to the interpretation of the effect of ramping limits. If there are no ramping limits, then the energy dispatch and energy prices of the co-optimized model would also be optimal for the model that excludes reserves and simply optimizes the energy dispatch with the scarcity price for reserves added as a constant to all the generation offers. But introduction of binding ramping limits would undo this simplicity.

One way to implement the two-step approximation is to assume different random draws for the two intervals from the distribution of net load change. Suppose that there are two variables y_I, y_{II} representing the incremental net load change in the two intervals. Further assume that the two variables have a common underlying distribution for a variable z but are proportional to the size of the interval. Then, assuming independence and with x the net load change over the full two intervals, we have:

$$E\left(y_I\right)=E\left(\delta z\right)=\delta E\left(z\right),$$
$$E\left(y_{II}\right)=E\left(\left(1-\delta\right)z\right)=\left(1-\delta\right)E\left(z\right).$$
$$Var\left(y_I\right)=Var\left(\delta z\right)=\delta^2 Var\left(z\right),$$
$$Var\left(y_{II}\right)=Var\left(\left(1-\delta\right)z\right)=\left(1-\delta\right)^2 Var\left(z\right).$$
$$E\left(z\right)=E\left(y_I+y_{II}\right)=E\left(x\right)=\mu.$$
$$Var\left(x\right)=Var\left(y_I+y_{II}\right)=Var\left(y_I\right)+Var\left(y_{II}\right)=\left(\delta^2+\left(1-\delta\right)^2\right)Var\left(z\right).$$
$$Var\left(z\right)=\frac{Var\left(x\right)}{\delta^2+\left(1-\delta\right)^2}=\frac{\sigma^2}{\delta^2+\left(1-\delta\right)^2}.$$

The implied variance of the individual intervals is derived from the impact of the square root law for the standard deviation of the sums of independent random variables.

Hence, for the first interval, the standard deviation is $\frac{\delta\sigma}{\sqrt{\delta^2+\left(1-\delta\right)^2}}$, where σ is the standard deviation of the net change in load over both intervals. With this adjustment, the revised version of (7) becomes:

$$P_R=v*\left(\delta*Lolp_I\left(r_R\right)+\left(1-\delta\right)*Lolp_{I+II}\left(r_R+r_{NS}\right)\right)=v*\delta*Lolp_I\left(r_R\right)+P_{NS},$$
$$P_{NS}=v*\left(1-\delta\right)*Lolp_{I+II}\left(r_R+r_{NS}\right). \tag{8}$$

Here the different distributions refer to the net change in load over the first interval, and over the sum of the two intervals. The distribution over the sum is just the same distribution for the whole period that was used above.

There would be an adjustment to deal with the minimum reserve to meet the max contingency. The revised formulation would include:

$$\pi_R\left(\hat{g}_R\right)=\begin{Bmatrix} Lolp_I\left(i^tK_R-\hat{g}_R-X\right), & i^tK_R-\hat{g}_R-X\geq 0 \\ 1, & i^tK_R-\hat{g}_R-X<0 \end{Bmatrix}$$
$$\pi_{NS}\left(\hat{g}_R\right)=\begin{Bmatrix} Lolp_{I+II}\left(i^tK_R-\hat{g}_R+i^tr_{NS}-X\right), & i^tK_R-\hat{g}_R+i^tr_{NS}-X\geq 0 \\ 1, & i^tK_R-\hat{g}_R+i^tr_{NS}-X<0 \end{Bmatrix}$$
$$\hat{P}_R\left(\hat{g}_R\right)=v*\left(\delta*\pi_R\left(\hat{g}_R\right)+\left(1-\delta\right)*\pi_{NS}\left(\hat{g}_R\right)\right),$$
$$\hat{P}_{NS}=v*\left(1-\delta\right)*\pi_{NS}\left(\hat{g}_R\right).$$

Returning to the approximation of simultaneous co-optimization of energy and Reserves with an ORDC, the key connection is in the design of the function $\hat{P}_R\left(\hat{g}_R\right)$, derived from the ORDC. Recall that with $\nu = VOLL - \partial\hat{C}_R$ and $\hat{P}_R\left(i^t g_R\right) = P_R\left(i^t r_R = i^t\left(K_R - g_R\right)\right)$. Everything else would stay the same in the approximating model, with the optimal level of reserves determining the scarcity opportunity cost of responsive generation as:

$$GENROP\left(\hat{g}_R\right) = \int_0^{\hat{g}_R} \hat{P}_R\left(x\right)dx = \int_0^{\hat{g}_R}\left(VOLL - \partial\hat{C}_R\left(\hat{g}_R\right)\right)*\left(\delta*\pi_R\left(x\right)+\left(1-\delta\right)*\pi_{NS}\left(x\right)\right)dx.$$

The resulting dispatch model, the approximation of equation (2) would be.

$$\begin{array}{llll}
 & \underset{d,g_R,g_{NR},\hat{g}_R\geq 0;y}{Max} \quad B\left(d\right)-C_R\left(g_R\right)-GENROP\left(\hat{g}_R\right)-C_{NR}\left(g_{NR}\right) & & \\
 & d-g_R-g_{NR}=y & \text{Net Loads} & \rho \\
 & i^t y = 0 & \text{Load Balance} & \lambda \\
(9) & Hy \leq b & \text{Transmission Limits} & \mu \\
 & g_R \leq K_R & \text{Responsive Capacity} & \theta_R \\
 & g_{NR} \leq K_{NR} & \text{Generation Only Capacity} & \theta_{NR} \\
 & i^t g_R = \hat{g}_R & \text{Responsive Generation Aggregation} & \xi.
\end{array}$$

This formulation ignores the second order impacts of the effect on reserve prices.

7. Endnote

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New York Independent System Operator, New York Power Pool, New York Utilities Collaborative, Niagara Mohawk Corporation, NRG Energy, Inc., Ontario Attorney General, Ontario IMO, Ontario Ministries of Energy and Infrastructure, Pepco, Pinpoint Power, PJM Office of Interconnection, PJM Power Provider (P3) Group, Powerex Corp., PPL Corporation, PPL Montana LLC, PPL EnergyPlus LLC, Public Service Company of Colorado, Public Service Electric & Gas Company, Public Service New Mexico, PSEG Companies, Red Wolf Energy Trading, Reliant Energy, Rhode Island Public Utilities Commission, San Diego Gas & Electric Company, Sempra Energy, SESCO LLC, Shell Energy North America (U.S.) L.P., SPP, Texas Genco, Texas Utilities Co, Twin Cities Power LLC, Tokyo Electric Power Company, Toronto Dominion Bank, Transalta, TransAlta Energy Marketing (California), TransAlta Energy Marketing (U.S.) Inc., Transcanada, TransCanada Energy LTD., TransÉnergie, Transpower of New Zealand, Tucson Electric Power, Westbrook Power, Western Power Trading Forum, Williams Energy Group, Wisconsin Electric Power Company, and XO Energy. The views presented here are not necessarily attributable to any of those mentioned, and any remaining errors are solely the responsibility of the author. (Related papers can be found on the web at www.whogan.com).